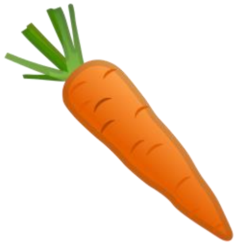

The Effectiveness of Subsidies and Taxes in Atomic Congestion Games

Bryce L. Ferguson[1], Philip N. Brown[2], Jason R. Marden[1]

[1] University of California, Santa Barbara department of Electrical and Computer Engineering

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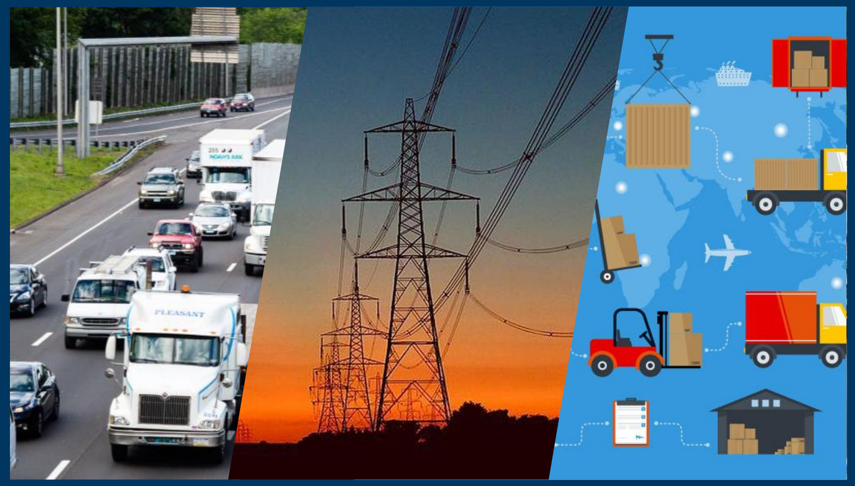
Socio-Technical Systems

Social-system

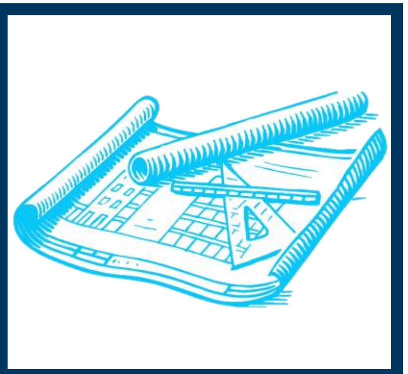
- Interconnected Users
- Diverse Population
- Individual Objectives

Socio-Technical system



- Interaction of ***users*** and ***infrastructure***
- Societal performance

Engineered-system



- Large-scale infrastructure
- Roads, power-grids, transport network, etc.

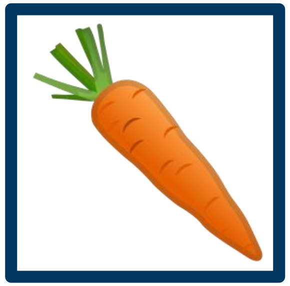
Goal: **Incentivize users** to promote efficient use of the infrastructure.

Carrots vs Sticks

Taxes/Tolls
(+)
Added cost to users

Tolls

Subsidies/Rebates
(-)
Reduced cost to users



Subsidies

Transportation

A. De Palma, R. Lindsey, “Private roads, competition, and incentives to adopt time-based congestion tolling,” *Elsevier*

P. Maill´e and N. E. Stier-Moses, “Eliciting Coordination with Rebates,” *Transportation Science*

Power Grids

Q. Wang, M. Liu, R. Jain, “Dynamic pricing of power in smart-grid networks,” *IEEE Conference on Decision and Control*

S. Huang, Q. Wu, “Dynamic Tariff-Subsidy Method for PV and V2G Congestion Management in Distribution Networks,” *IEEE Transactions on Smart Grid*

Supply-chain Management

M. Christopher, J. Gattorna, “Supply chain cost management and value-based pricing,” *Elsevier*

T. A. Taylor, “Supply Chain Coordination Under Channel Rebates with Sales Effort Effects,” *Tech. Rep.*

Carrots vs Sticks

Taxes/Tolls

(+)

Added cost to users

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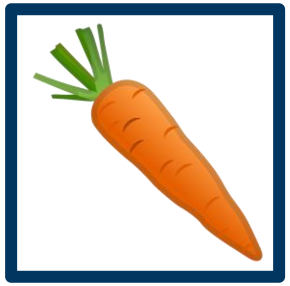


Subsidies

Both are viable methods of influencing users in many settings

Both can be implemented with similar technology/infrastructure

Both can be monetarily feasible (fees vs reimbursements to up front cost)

Q?: What are the capabilities of each incentive?

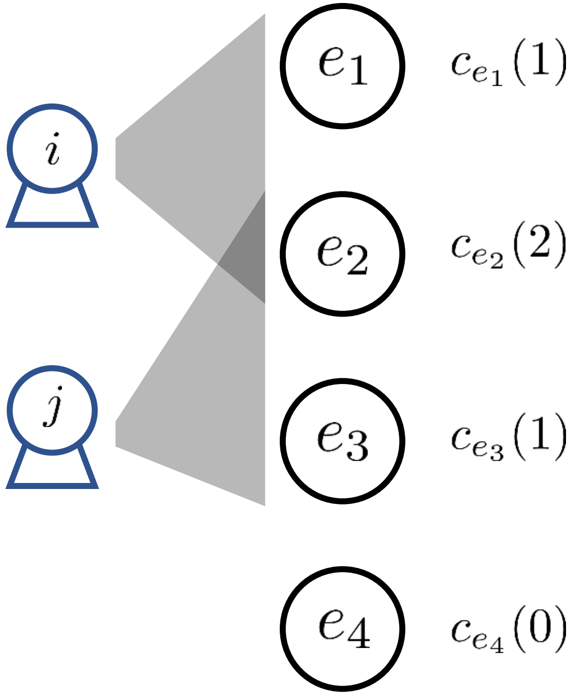
Model

- Congestion Game G
  - Resources $\mathcal{E} = \{1, \dots, E\}$
  - Agents $N = \{1, \dots, n\}$
  - Actions $a_i \in \mathcal{A}_i \subset 2^{\mathcal{E}}$
    - Allocation $a = (a_1, \dots, a_n) \in \mathcal{A}$
  - Cost functions $c_e(|a|_e) \geq 0$
  - Total Cost

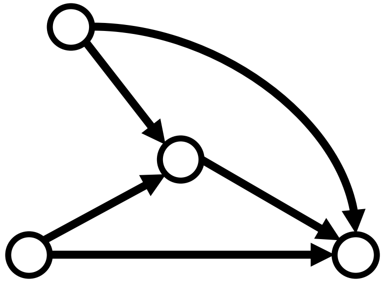
$$C(a) = \sum_{e \in \mathcal{E}} |a|_e c_e(|a|_e)$$

  - Optimal allocation

$$a^{\text{opt}} \in \arg\min_{a \in \mathcal{A}} C(a)$$



Eg:
Network
Congestion



Selfish Decision Making

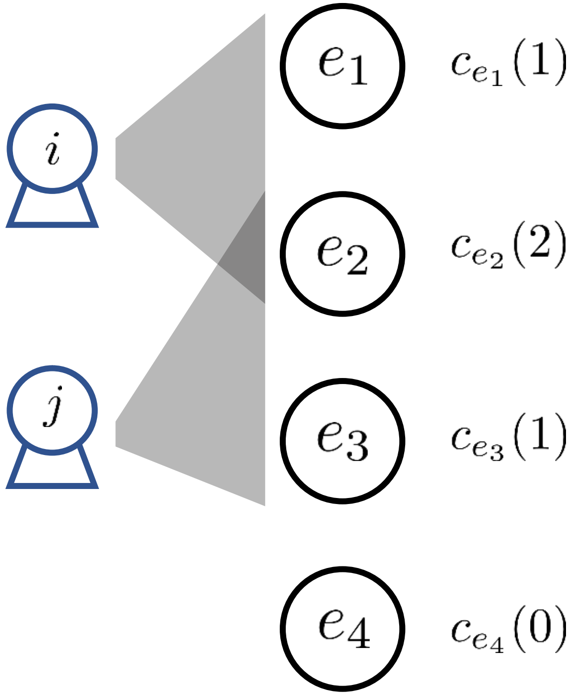
What happens when users choose their own routes?

Cost of user i

$$J_i(a_i, a_{-i}) = \sum_{e \in a_i} c_e(|a|_e)$$

Equilibrium: Nash equilibrium a^{Ne}

$$a_i^{\mathrm{Ne}} \in \arg\min_{a_i \in \mathcal{A}_i} \sum_{e \in a_i} c_e(|a|_e^{\mathrm{Ne}}) \quad \forall i \in N$$



Price of Anarchy

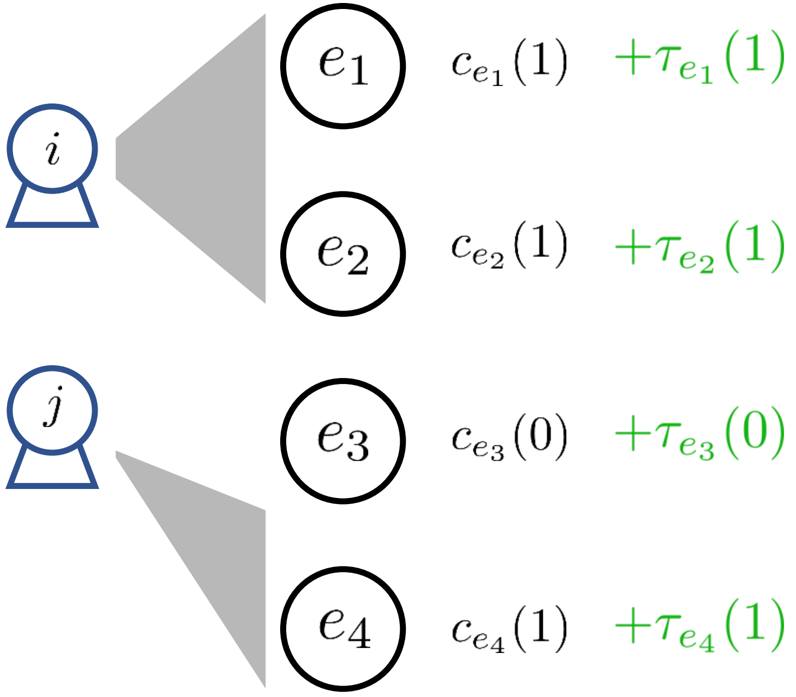
$$\mathrm{PoA}(G) := \frac{\max_{a^{\mathrm{Ne}} \in \mathrm{NE}(G)} C(a^{\mathrm{Ne}})}{C(a^{\mathrm{opt}})} \geq 1$$

How to reduce this inefficiency?

Incentive Mechanism: $T(c_e) = \tau_e$

Selfish Decision Making

What happens when users choose their own routes?



Cost of user i

$$J_i(a_i, a_{-i}) = \sum_{e \in a_i} c_e(|a|_e) + \tau_e(|a|_e)$$

Equilibrium: Nash equilibrium a^{Ne}

$$a_i^{\mathrm{Ne}} \in \arg\min_{a_i \in \mathcal{A}_i} \sum_{e \in a_i} \left(c_e(|a|_e^{\mathrm{Ne}}) + \tau_e(|a|_e^{\mathrm{Ne}})\right) \quad \forall i \in N$$

Price of Anarchy

$$\mathrm{PoA}(G, T) := \frac{\max_{a^{\mathrm{Ne}} \in \mathrm{NE}(G,T)} C(a^{\mathrm{Ne}})}{C(a^{\mathrm{opt}})} \geq 1$$

How to reduce this inefficiency?

Incentive Mechanism: $T(c_e) = \tau_e$

Incentives: Taxes & Subsidies

Taxes

Tax function:

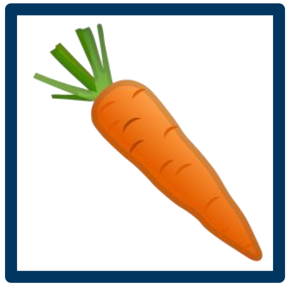
$$\tau_e^+(x) \geq 0 \quad \forall x \geq 0$$

Taxation mechanism:

$T^+(c_e) = \tau_e^+$ Only assigns tolls

Optimal taxation mechanism:

$$T^{\text{opt}+} \in \arg\min_{T^+} \text{PoA}(G, T^+)$$



Subsidies

Subsidy function:

$$\tau_e^-(x) \leq 0 \quad \forall x \geq 0$$

Subsidy mechanism:

$T^-(c_e) = \tau_e^-$ Only assigns subsidies

Optimal subsidy mechanism:

$$T^{\text{opt}-} \in \arg\min_{T^-} \text{PoA}(G, T^-)$$

Tolls

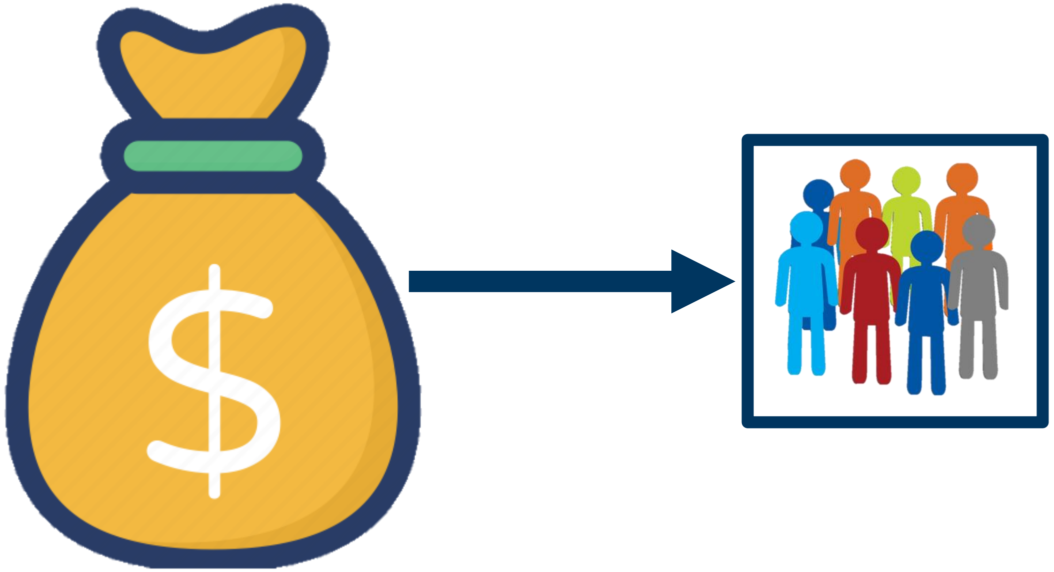
$\text{PoA}(G, T^{\text{opt}+})$

Subsidies

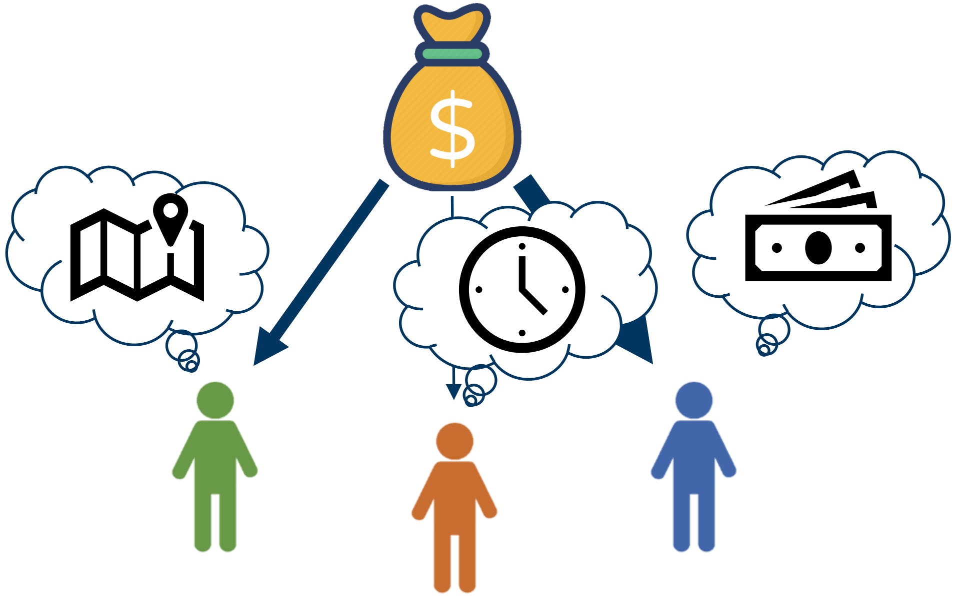
$\text{PoA}(G, T^{\text{opt}-})$

Challenges

Budgetary constraints



User heterogeneity

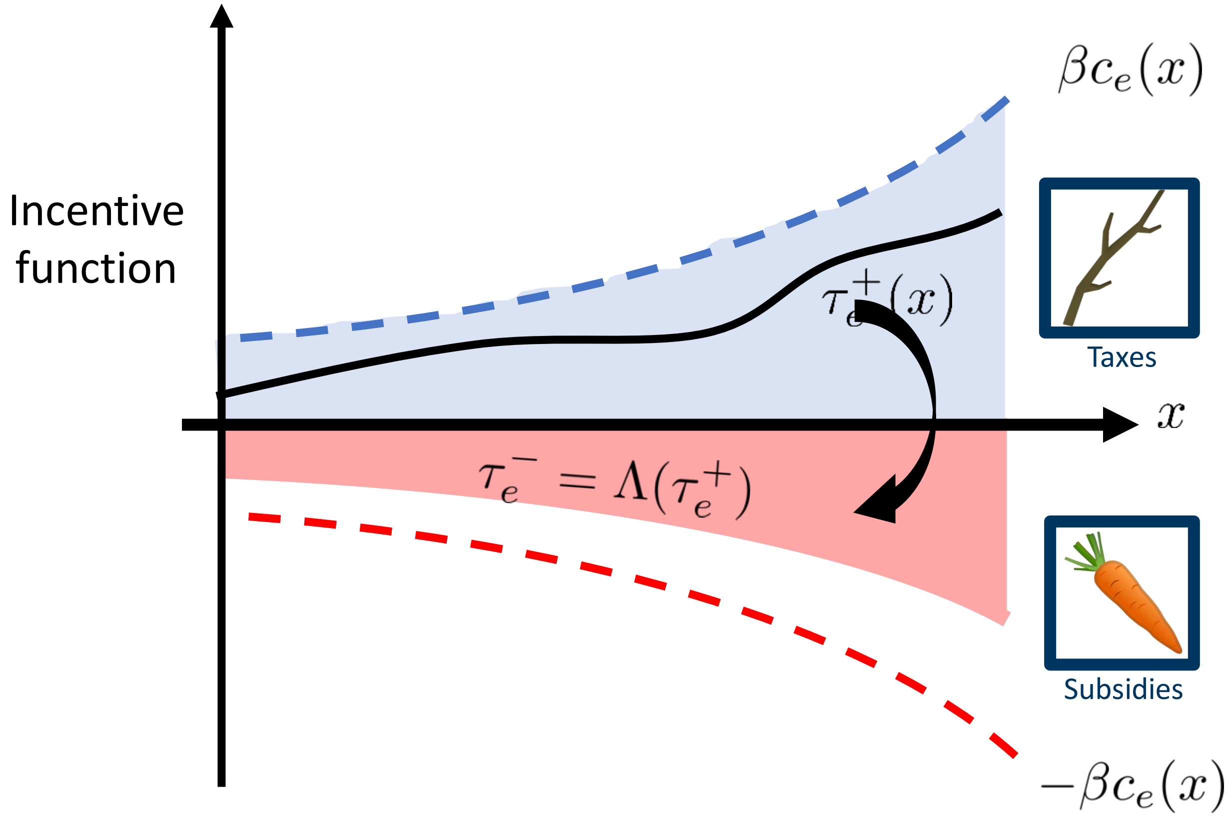


Taxes

Subsidies

Budgetary Constraints

Added Constraint: $|\tau_e(x)| \leq \beta c_e(x) \quad \forall x \geq 0$



Theorem 1

For a congestion games G, under bounding factor $\beta \geq 0$,

$$\mathrm{PoA}(G, T^{\mathrm{opt}+}(\beta)) \geq \mathrm{PoA}(G, T^{\mathrm{opt}-}(\beta)) \geq 1.$$

Additionally, if the budget constraint is active for every optimal incentive, the inequalities are strict.

Smaller subsidies can outperform ***larger taxes***.

User Heterogeneity

Each user has unknown price-sensitivity $s_i \in [S_L, S_U]$

$$J_i(a_i, a_{-i}) = \sum_{e \in a_i} c_e(|a|_e) + s_i \tau_e(|a|_e)$$

$0 < S_L/S_U \leq 1$
is a measure of uncertainty

Robust performance guarantee:

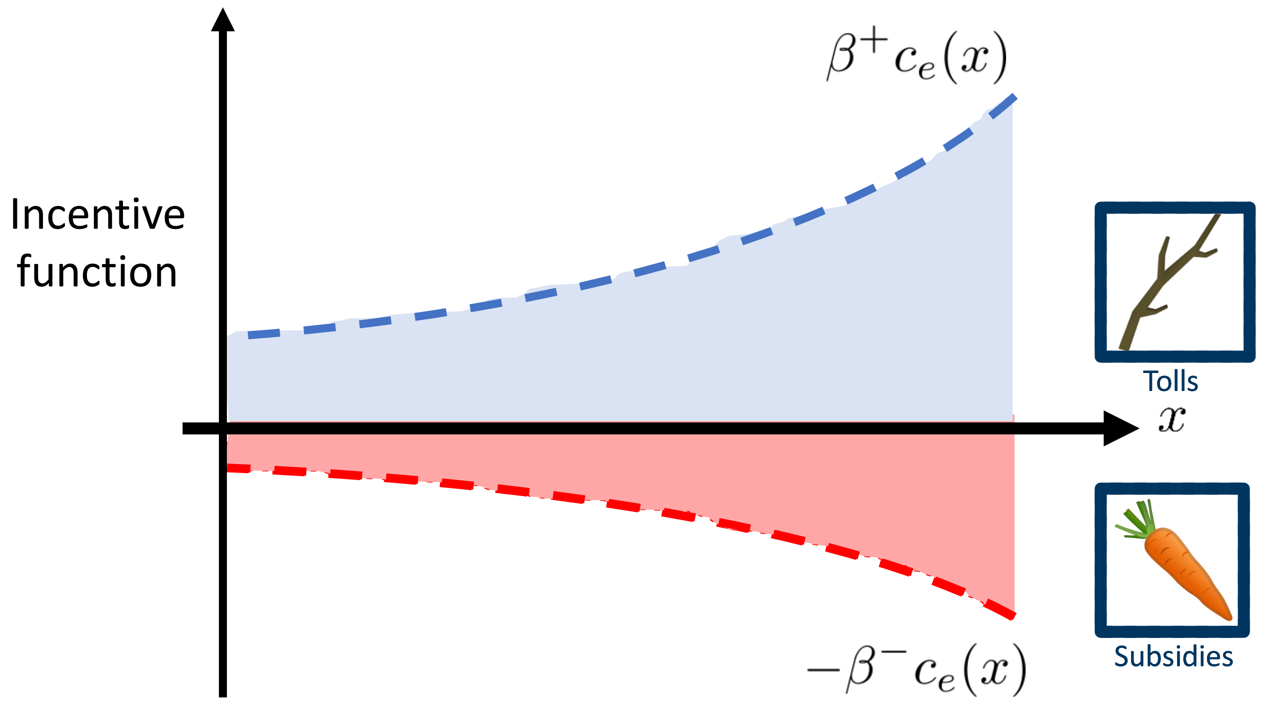
$$\mathrm{PoA}(G, \mathcal{S}, T) = \sup_{s \in \mathcal{S}} \frac{\max_{a^{\mathrm{Ne}} \in \mathrm{NE}(G,s,T)} C(a^{\mathrm{Ne}})}{C(a^{\mathrm{opt}})}$$

Q?: How do incentives perform with user heterogeneity?

Budgetary Constraints & User Heterogeneity

Start with *nominally equivalent* bounded subsidies and tolls, i.e.,

$\mathrm{PoA}(G, T^{\mathrm{opt}+}(\beta^+)) = \mathrm{PoA}(G, T^{\mathrm{opt}-}(\beta^-))$ when users are homogeneous.



As user become heterogeneous:

Theorem 2

For a congestion games G, under bounding factors $\beta^+,\ \beta^-$ respectively, with possible price-sensitivity distributions $\mathcal{S}$,

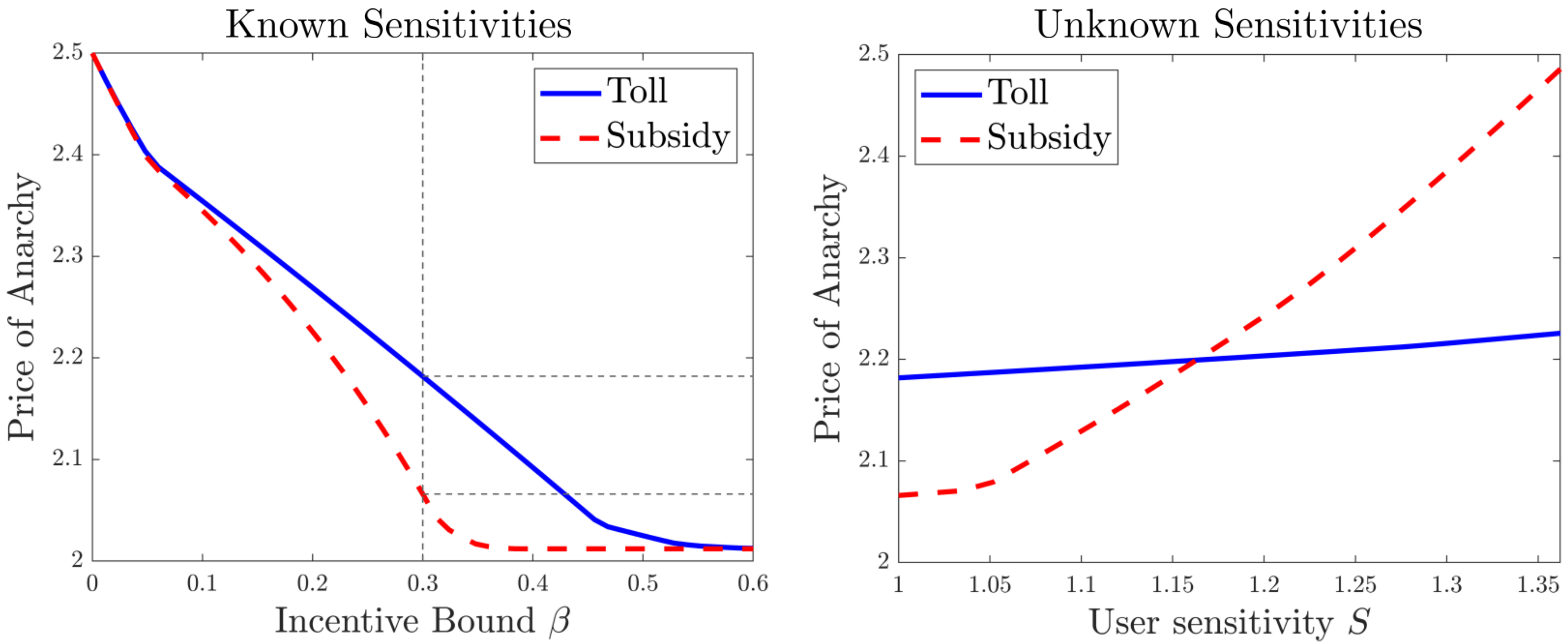
$$\mathrm{PoA}(G, \mathcal{S}, T^{\mathrm{opt}-}(\beta^-, \mathcal{S})) \geq \mathrm{PoA}(G, \mathcal{S}, T^{\mathrm{opt}+}(\beta^+, \mathcal{S})) \geq 1.$$

Additionally, if G is responsive to user heterogeneity, the inequalities are strict.

Performance of ***subsidies is less robust*** to player heterogeneity than taxes.

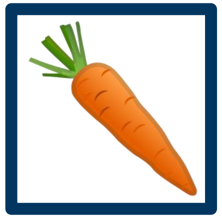
Computational Example

Price of anarchy bound over affine congestion games.



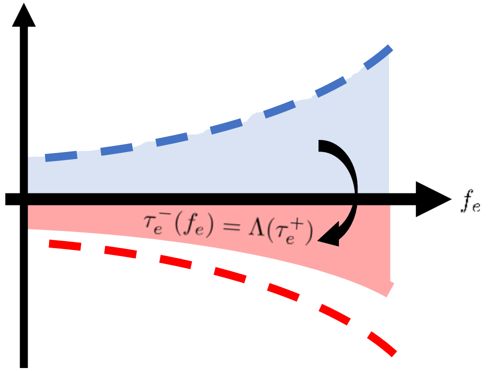
Conclusion

- Low heterogeneity



Subsidies

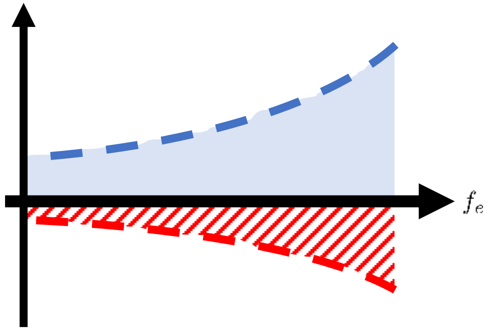
Thm. 1



- High heterogeneity

Taxes

Thm. 2



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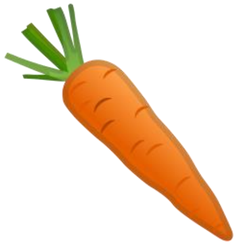

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